hw set 24
sect 8.5
p 598

4. $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n+1}$

$\lim_{n\to\infty} \left| \frac{x^{n+1}}{(n+1)+1} \cdot \frac{n+1}{x^n} \right| = \lim_{n\to\infty} \left| \frac{x(n+1)}{(n+2)} \right|$

$= |x| \lim \left(\frac{n+1}{n+2}\right) = |x| < 1$

$\boxed{R = 1}$

$x=1$, $\sum \frac{(-1)^n}{n+1}$ Aternating

$\lim_{n\to\infty} \frac{1}{n+1} \Rightarrow 0$

$\therefore$ converges

$x=-1$, $\sum \frac{(-1)^n(-1)^n}{n+1} = \sum \frac{1}{n+1}$

diverges

$\boxed{\therefore \text{ interval is } (-1, 1]}$

5. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^n}{n^3}$

$\lim_{n\to\infty} \left| \frac{(-1)^n x^{n+1}}{(n+1)^3} \cdot \frac{n^3}{(-1)^{n-1} x^n} \right|$

$= \lim_{n\to\infty} \left| \frac{-x \cdot n^3}{n^3+3n^2+3n+1} \right|$

$= |x| \lim_{n\to\infty} \left(\frac{n^3}{n^3+3n^2+3n+1} \right) = |x| < 1$

$\boxed{R = 1}$

$x=1$, $\sum \frac{(-1)^{n-1}}{n^3}$ Alternating

$\lim_{n\to\infty} \frac{1}{n^3} = 0$ $\therefore$ converges

$x=-1$, $\sum \frac{(-1)^{n-1}(-1)^n}{n^3} = \sum \frac{(-1)^{2n-1}}{n^3} = \sum \frac{-1}{n^3}$

converges

$\boxed{\therefore \text{ interval is } [-1, 1]}$

6. $\sum_{n=1}^{\infty} \sqrt{n}\, x^n$

$\lim_{n\to\infty} \left| \frac{\sqrt{n+1}\, x^{n+1}}{\sqrt{n}\, x^n} \right|$

$= \lim_{n\to\infty} \sqrt{\frac{n+1}{n}} \cdot |x| = |x| < 1$

$\boxed{R = 1}$

at $x=1$, $\sum \sqrt{n}$ diverges

at $x=-1$, $\sum \sqrt{n}(-1)^n$ Alternating

$\lim_{n\to\infty} \sqrt{n} = \infty$

$\therefore$ diverges

$\boxed{\text{interval is } (-1, 1)}$

7. $\sum_{n=0}^{\infty} \frac{x^n}{n!}$

$\lim_{n\to\infty} \left| \frac{x^{n+1}}{(n+1)!} \cdot \frac{n!}{x^n} \right|$

$= \lim_{n\to\infty} \left| \frac{x}{n+1} \right| = |x| \lim_{n\to\infty} \frac{1}{n+1} = 0$

$R = \infty$

$(-\infty, \infty)$

8. $\sum_{n=1}^{\infty} \frac{x^n}{n3^n}$

$$\lim_{n\to\infty} \left| \frac{x^{n+1}}{(n+1)3^{n+1}} \cdot \frac{n3^n}{x^n} \right|$$

$$= \lim_{n\to\infty} \left| \frac{xn}{3(n+1)} \right|$$

$$= |x| \lim \left(\frac{n}{3n+3}\right) = \frac{|x|}{3} < 1$$

So $|x| < 3$

$\boxed{R=3}$

$x=3$, $\sum \frac{3^n}{n3^n} = \sum \frac{1}{n}$ diverges

$x=-3$, $\sum \frac{(-3)^n}{n3^n} = \sum \frac{(-1)^n}{n}$ Aternating

$\lim_{n\to\infty} \frac{1}{n} = 0$ $\therefore$ Converg.

$\boxed{\text{Interval is } [-3,3)}$

9. $\sum_{n=1}^{\infty} \frac{(-2)^n x^n}{\sqrt[4]{n}}$

$$= \lim_{n\to\infty} \left| \frac{(-2)^{n+1} x^{n+1}}{(n+1)^{1/4}} \cdot \frac{n^{1/4}}{(-2)^n x^n} \right|$$

$$= \lim_{n\to\infty} \left| \frac{-2xn^{1/4}}{(n+1)^{1/4}} \right|$$

$$= 2|x| \lim_{n\to\infty} \sqrt[4]{\frac{n}{n+1}} = 2|x| < 1$$

$\therefore$ $|x| < \frac{1}{2}$ $\boxed{R=\frac{1}{2}}$

at $x=\frac{1}{2}$,

$$\sum \frac{(-2)^n(\frac{1}{2})^n}{\sqrt[4]{n}} = \sum \frac{(-1)^n}{\sqrt[4]{n}}$$

Alternating:

$$\lim_{n\to\infty} \frac{1}{\sqrt[4]{n}} = 0$$

$\therefore$ converges

at $x=-2$,

$$\sum \frac{(-2)^n(-\frac{1}{2})^n}{\sqrt[4]{n}} = \sum \frac{1}{\sqrt[4]{n}}$$ ~~converges~~ diverges

~~Since~~ p-test $1/4 \leq 1$

$\boxed{\therefore \text{ interval is } (-\tfrac{1}{2},\tfrac{1}{2}]}$

10. $\sum_{n=1}^{\infty} \frac{x^n}{5^n n^5}$

$$\lim_{n\to\infty} \left| \frac{x^{n+1}}{5^{n+1}(n+1)^5} \cdot \frac{5^n n^5}{x^n} \right|$$

$$= \lim_{n\to\infty} \left| \frac{xn^5}{5(n+1)^5} \right| = \frac{|x|}{5} \lim_{n\to\infty} \left(\frac{n}{n+1}\right)^5 = \frac{|x|}{5} < 1$$

So $|x| < 5$ $\boxed{R=5}$

at $x=5$, $\sum \frac{(5)^n}{5^n n^5} = \sum \frac{1}{n^5}$ converges

at $x=-5$, $\sum \frac{(-5)^n}{5^n n^5} = \sum \frac{(-1)^n}{n^5}$ Alternating

$\lim_{n\to\infty} \frac{1}{n^5} = 0$ converges

$\boxed{\text{interval is } [-5,5]}$

12. $\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$

$$\lim_{n\to\infty} \left| \frac{(-1)^{n+1} x^{2(n+1)}}{[2(n+1)]!} \cdot \frac{(2n)!}{(-1)^n x^{2n}} \right|$$

$$= \lim_{n\to\infty} \left| \frac{x^{2n+2}}{(2n+2)!} \frac{(2n)!}{x^{2n}} \right|$$

$$= \lim_{n\to\infty} \left| \frac{x^2}{(2n+2)(2n+1)} \right| = 0$$

So $R = \infty$

& $(-\infty, \infty)$

14. $\sum_{n=1}^{\infty} \frac{(-2)^n}{\sqrt{n}} (x+3)^n$

$$\lim_{n\to\infty} \left| \frac{(-2)^{n+1}(x+3)^{n+1}}{\sqrt{n+1}} \cdot \frac{\sqrt{n}}{(-2)^n (x+3)^n} \right|$$

$$= \lim_{n\to\infty} \left| \frac{-2(x+3)\sqrt{n}}{\sqrt{n+1}} \right|$$

$$= 2|x+3| \lim_{n\to\infty} \sqrt{\frac{n}{n+1}} = 2|x+3| < 1$$

so, $|x+3| < 1/2$

$\boxed{R = 1/2}$

$-1/2 < x+3 < 1/2$

$-7/2 < x < -5/2$

at $x = -7/2$,

$\sum \frac{(-2)^n}{\sqrt{n}} \cdot (-\frac{1}{2})^n = \sum \frac{1}{\sqrt{n}}$ diverges

at $x = -5/2$

$\sum \frac{(-2)^n}{\sqrt{n}} \cdot (\frac{1}{2})^n = \sum \frac{(-1)}{\sqrt{n}}$ Converges

$\boxed{\text{interval is } [-7/2, -5/2]}$

16. $\sum_{n=1}^{\infty} \frac{n(x-4)^n}{n^3+1}$

$$\lim_{n\to\infty} \left| \frac{(n+1)(x-4)^{n+1}}{(n+1)^3+1} \cdot \frac{n^3+1}{(n)(x-4)^n} \right|$$

$$= \lim_{n\to\infty} \left| \frac{(x-4)(n+1)(n^3+1)}{n(n+1)^3+n} \right|$$

$$= |x-4| \lim_{n\to\infty} \left| \frac{n^4+n+n^3+1}{n^4+3n^3+3n^2+2n} \right|$$

$= |x-4| < 1$

So, $\boxed{R = 1}$

$-1 < x-4 < 1$

$3 < x < 5$

at $x = 3$, $\sum \frac{n(-1)^n}{n^3+1}$ Alternating $\lim_{n\to\infty} \frac{n}{n^3+1} = 0$ $\therefore$ converges

at $x = 5$, $\sum \frac{n}{n^3+1} \le \sum \frac{n}{n^3} = \sum \frac{1}{n^2}$ converges

$\boxed{\text{interval is } [3, 5]}$